Springs, Shocks and your Suspension

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Understanding how your springs and shocks move as your race car moves through its range of motions is one of the basics you must understand to make your spring and shock choices properly. Springs and shock absorber damping curves are all measured at a one-to-one ratio. What this means is one inch of travel on the spring provides a load value for a spring, the second inch of travel provides a new load value for the spring etc. The load and the displacement are then plotted and the ratio between the load value and the displacement value provides the "Spring rate". Your race car doesn't care what the spring rate is, it is the rate at the tire called the wheel rate, that provides the stiffness that supports the car and determines the cars handling characteristics. When the springs and shocks are installed in your race car they are rarely installed in a one-to-one ratio. They are installed possibly on an arm that is connected to the axle or suspension at some distance from the wheel/axle centerline. The difference between how much the wheel moves up and down and how much the spring compresses is called the SPRING MOTION RATIO (MR_{SP}). For the shock absorber the difference between how much the wheel moves up and down and how much the shock compresses (or extends) is called the SHOCK MOTION RATIO (MR_{SH}). On a coil-over spring-shock system the MR_{Sp} and the MR_{Sh} are equal.

Wheel forces vs. spring forces

The motion ratio describes the ratio between the vertical displacement at the centerline of the wheel and the spring compression. The motion ratio alters the force the spring itself exerts and the force exerted at the wheel centerline. In addition the motion ratio alters the displacement (compression) the spring experiences to the displacement of the wheel centerline. Since both the force is altered and the displacement is altered a small motion ratio can dramatically alter the relationship between the stiffness of the installed spring and the displacement stiffness at the wheel. The spring rate at the wheel, called the wheel rate, and the actual rate of the spring, called the spring rate are related through the motion ratio. This is shown in Figure 1 below.

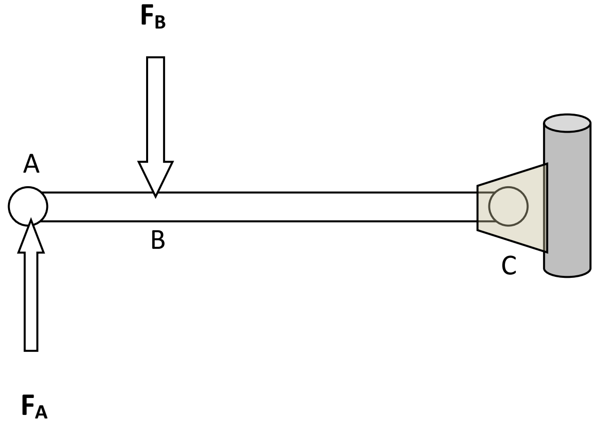

Figure 1: Forces at points on a rotating arm

Just as in a common lever system such as a crowbar, the force required at location B (F_B) is greater than the force exerted at location A (F_A). The difference in the forces is dependent on the distance between

A and C ($L_{A\text{-}C}$), and the distance between B and C ($L_{B\text{-}C}$). This is also true for the speeds at A and at B. The difference in the speed of travel is dependent on the distance between A and C ($L_{A\text{-}C}$), and the distance between B and C ($L_{B\text{-}C}$). The actual formula is shown below.

$$F_B = F_A \frac{L_{(A-C)}}{L_{(B-C)}}$$ **Equation 1**

Let's take a simple example of a 16 inch arm length from pivot to pivot. Let's also say the length from where the spring mounts to the inner pivot is 12 inches. The force at B (F_B) is then,

$$F_B = F_A \frac{L_{(A-C)}}{L_{(B-C)}} = F_A \times \frac{16}{12} = 1.33 F_A$$ **Equation 2**

It could also be looked at from the wheel side (A) as shown in Equation 3.

$$F_A = F_B \frac{L_{(B-C)}}{L_{(A-C)}} = F_B \times \frac{12}{16} = 0.75\ F_B$$ **Equation 3**

Note that the force required at B is 1.33 times greater than the force at A. This tells you the spring, if mounted at B, is required to exert 1.33 times as much force as would be required at A to hold up the race car or approximately 1.33 times the scale weight on that tire.

Motion Ratio

The spring motion ratio (MR_{Sp}) is defined as the ratio of the spring displacement (d_B) to the wheel displacement (d_A), as shown in Equation 4 below. Since the arm rotates around a common pivot, the length between the pivot and the points of interest also determine the motion ratio. In our case the distance measured between A & C and the distance between B & C. Notice in Figure 2, the displacement at B is less than the displacement at A. This ratio is the motion ratio as shown in Equation 4. Note that the closer point B moves to point C the less the displacement of B, for any displacement at A, so the displacement is directly related to the distance on the arm from the pivot.

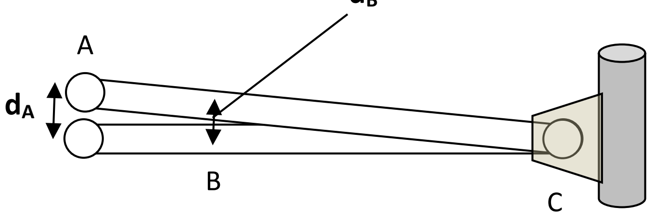

Figure 2: **Displacement at points on a rotating arm**

Mathematically the motion ratio (MR) is defined by the displacement ratio or the length ratio as shown in Equation 4.

$$MR = \frac{d_B}{d_A} = \frac{L_{(B-C)}}{L_{(A-C)}} \qquad \textbf{Equation 4}$$

For our previous example with a 16 inch arm and point B located 12 inches from the pivot, the motion ratio is as shown in Equation 5.

$$MR = \frac{d_B}{d_A} = \frac{L_{(B-C)}}{L_{(A-C)}} = \frac{12}{16} = 0.75:1 \qquad \textbf{Equation 5}$$

Equation 5 rearranged also indicates the displacement at A as shown in Equation 6, for our example. The displacement at B, relative to the displacement at A is shown in Equation 7.

$$d_A = \frac{d_B}{MR} = \frac{d_{(B)}}{0.75} = 1.33 \times d_B \qquad \textbf{Equation 6}$$

$$d_B = MR \times d_A = 0.75 \times d_A \qquad \textbf{Equation 7}$$

This is also true for the speeds at A and at B. The difference in the speed of travel at points A and B is dependent on the distance between A and C ($L_{A\text{-}C}$), and the distance between B and C ($L_{B\text{-}C}$). Motion ratio therefore is very important for selecting both spring rates and for shock valving. In terms of shock valving the damping force is dependent upon how quickly the shock compresses (compression damping) and extends (rebound damping). *A word of caution, some Supermodifieds are designed with shocks that shorten during rebound and lengthen during suspension compression.* The motion ratio and the speed ratio (SR), if the shock is at the same location as the spring, are the same.

$$SR = MR = \frac{d_B}{d_A} = \frac{L_{(B-C)}}{L_{(A-C)}} = \frac{12}{16} = 0.75:1 \qquad \textbf{Equation 8}$$

If the force and the displacement are examined, the force at B is greater, and the displacement at B is less, when compared to the force and displacement at A. Springs are rated in lbs/in, or the force they generate per inch they are compressed. A 250 “pound” spring will require a force of 250 lbs. to compress 1 inch and 500 lbs. to compress 2 inches etc. Therefore the force required at a given displacement or force/displacement is the spring rate. This is shown in Figure 3.

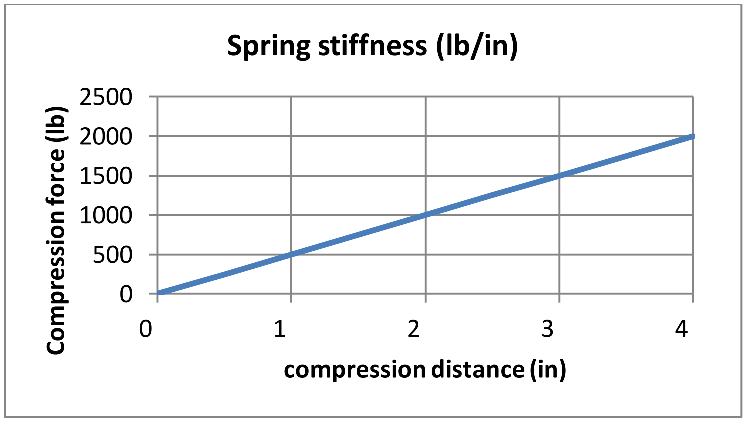

Figure 3: Spring rate plot for a 500 lb/in spring

From Equation 3 and Equation 5 the relationship between the forces at A and B for our example are determined.

$$F_A = MR_{Sp} \times F_B = 0.75\ F_B$$

Spring Rates and Wheel rates

Also the displacement relationship between A and B can be found as in Equation 6. If the actual spring relationships are compared at A and at B, forces and the displacements form the basis for the spring rate and wheel rate as indicated in Equation 9.

$$Spring\ rate = \frac{F}{d}$$

$$Spring\ rate_A = \frac{F_A}{d_A} = \frac{MR_{Sp} \times F_B}{\frac{d_B}{MR_{Sp}}} = MR_{Sp} \times MR_{Sp} \times \frac{F_B}{d_B}$$

Equation 9

$$Spring\ rate_A = MR_{Sp}{}^2 \times Spring\ rate_B$$

This means the spring rate at A, in our example, is as shown in Equation 10.

$$Spring\ rate_A = MR_{Sp}{}^2 \times Spring\ rate_B$$

$$Spring\ rate_A = .75 \times .75 \times Spring\ rate_B = 0.56 \times Spring\ rate_B$$

Equation 10

This means a 500 lb/in spring would actually only be acting as a 280 lb/in spring at the wheel as shown in Figure 4.

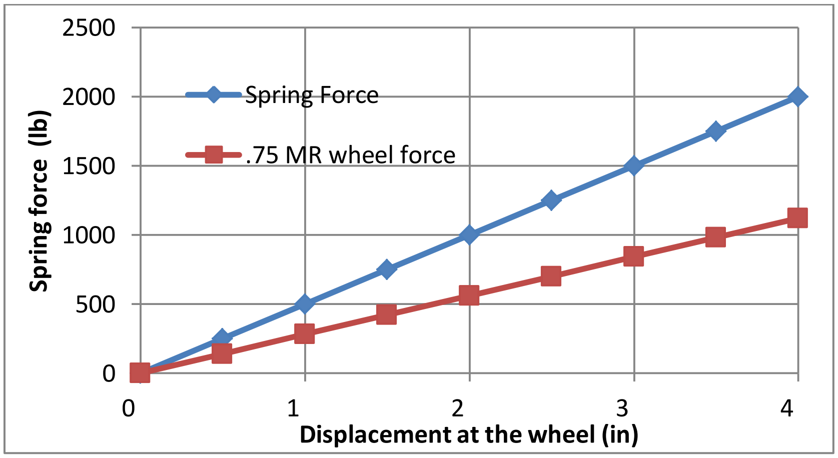

Figure 4: Spring stiffness and Suspension stiffness for our example

The importance of this is to understand it is the motion ratio multiplied by itself or "squared" which determines the spring rates to be used as shown in Equation 11 (This is for our example).

$$Spring\ rate_B = \frac{Desired\ wheel\ rate}{MR_{Sp}{}^2}$$

Equation 11

$$Spring\ rate_B = \frac{Desired\ Wheel\ rate}{0.75 \times 0.75} = 1.78 \times\ Desired\ wheel\ rate$$

Inclined spring axis or angled arms

Figure 5 shows an independent suspension with the spring and shock absorber inclined to the inside. The incline results in an angle (α) between the spring axis and the line perpendicular to the lower control arm plane (connecting the inner and outer heim joint balls). To improve the accuracy of Equation 10 and 11 further, the motion ratio needs to reflect any angle that exists between the arm and the spring as indicated in Equation 12. For your convenience I have included the Cosine values for the corresponding angles in Table 1 which also shows small angles do not greatly affect the wheel rates.

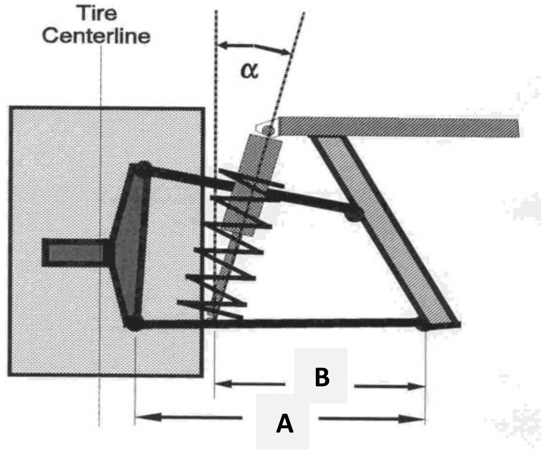

Figure 5: Independent suspension with an inclined spring-shock axis.

$$MR = \frac{d_B}{d_A} \times \text{cosine}(\alpha) = \frac{L_{(B-C)}}{L_{(A-C)}} \times \text{cosine}(\alpha)$$

Equation 12

ANGLE (α)	Cosine(α)	ANGLE (α)	Cosine(α)
0	1.00	25	0.906
5	0.996	30	0.866
10	0.985	35	0.819
15	0.966	40	0.766
20	0.940	45	0.707

Table 1: Measured angle and corresponding Cos values

If the axis is inclined 10 degrees from the plane of the lower control arm the motion ratio now becomes as indicated in Equation 13 and the spring rate and wheel rate are related through Equation 14.

$$SR = MR = \frac{d_B}{d_A} \times \cos(\alpha) = \frac{L_{(B-C)}}{L_{(A-C)}} \times \cos(\alpha) = \frac{12}{16} \times .985 = 0.739:1$$

Equation 13

$$Wheel\ rate = MR_{Sp}^{\ 2} \times Spring\ rate$$
$$Wheel\ rate_A = .739 \times .739 \times Spring\ rate = 0.542 \times Spring\ rate$$

Equation 14

As a design hint, if your independent suspension ends up with inclined arm, or your coil over is on an arm to which you are attaching a spring/shock mount you want to make sure the suspension, during compression, has a rising rate. The first step of setting this up is to raise the suspension to where it is at maximum travel, then place the coil-over top mount such that the spring axis and the control arm pivot plane result in an angle between them of 90 degrees or less (an angle α of 0 degrees). This will result in what is called a rising rate suspension (increasing stiffness as it goes up). If during the compression travel the angle between the coil over axis and the vertical perpendicular to the control arm approaches zero, and then passes through zero, you now have a rising then a falling rate as the suspension moves. The suspension will then have a stiffness that starts out increasing as it compresses and then begins decreasing as the travel increases. This is near impossible to get a consistent car throughout the corner. What is needed, as the suspension moves through its compression travel, is for the angle shown in Figure 3 to move toward zero, but never reverse itself to the opposite side of the coil-over axis line. If for instance the coil over is directly vertical and the control arm is flat, as the suspension moves the vertical from the arm moves inside of the coil-over axis and the motion ratio ends up decreasing which means the spring rate at the wheel is decreasing as the spring is compressing. If the vertical off the control arm plane stays outside of the coil-over axis as the suspension travels the rate is increasing but never reversing, a much more predictable situation. Excessively high angles will result in excessive spring rate gains which inherently cause push or looseness, depending on whether it is front or rear suspension.

Shock absorber speeds and shock dyno data considerations.

As shown in Equation 8 the speed ratio and the motion ratio are the same for a coil-over shock. If the shock is mounted on the arm inward of the wheel end pivot there is always a motion ratio that is less than 1.0. This motion ratio slows down the shock speed. There is nothing “wrong” with this except you need to know what speeds and damping forces you need to be looking at for your suspension. This means you should compare your shock dyno values at a different point on the curve than a car that has a 1:1 motion ratio. For instance in our previous example we calculated a motion ratio of 0.739 which results in a shock speed that is 73.9% of what is shown on the shock dyno sheet and 73.9% of the suspension movement speed. If for example you want to compare the damping at 3 inches/sec of wheel vertical movement you would need to move down the curve to 2.2 in/sec and examine the force there. However, now the force that is generated at 2.2 in/sec needs to be multiplied by 1.33 because the damping force needed at the wheel is now reduced by the motion ratio. A simple example might show a 100 lb. rebound damping force at 3 inches per second which is what you desire. However since you have a motion ratio of .739 and a force ratio of 1.33 you will need to examine your shock plot at 2.2 in/sec and you need to produce a rebound force of 1.33 x 100 or 133 lbs. Simply said, as you add motion ratio your shock speeds go down and your damping forces need to go up. Remember how as motion ratios decreased, the displacement at the spring went down and the forces at the spring had to increase. At the shock it is not much different except we are dealing with speed rather than displacement.

Measuring motion ratio

Now that we are through the math let's discuss how we can actually measure motion ratio. By measuring motion ratio all the angles and everything else is taken into account. When rockers and push links at high angles are used, many times measurement is the only answer. The recommended procedure follows.

1. With the car at ride height measure the angle of the arm, the length of the shock or the spring cup-to-cup spacing. This is your reference or "0" point.
2. Put the car up on jack stands and remove the coil spring and disconnect any anti-roll bars etc.
3. Place a jack under the suspension and move it to the ride height as measured in 1, the zero point. If it is a solid axle you may want to use two jacks to keep the axle level and to prevent bind on a 4-linked axle.
4. Measure and record the distance between the spring cups on the coil over (or the length of the shock). This is the zero point. (*Always do measurements in one direction so you are going to start by lowering the suspension and then raising it through zero, up to maximum travel.)*
5. Lower the suspension 2 inches and measure the distance between the spring cups (or the length of the shock) and record it.
6. Move the suspension up ½ inch and measure and record the new spring cup spacing. *(I like to do all measurement with a dial caliper for accuracy.)*
7. Repeat step 6 until you reach maximum compression travel.
8. Use graph paper or a computer to plot the results.

If for instance the first measurement taken at ½ inch of height above the starting point gives a change of shock length of .370 inch and when it is raised another ½ inch you measure a change of .380 the suspension is rising rate (increasing change) and the motion ratio is .370 + .380 or .750:1. If at 1.5 inches the change from the previous measurement is .390 and at 2.0 inches (ride height) the change from the previous is .395 you have a motion ratio of .785:1 which indicates still rising rate but it appears you are approaching a falling rate since the second reading (.395) is a smaller increase than the one taken at 1 ½ inches from the bottom. If in the next ½ inch the change is .380 and the following 1/2 inch is .370 now you are into a falling rate. I don't like a falling rate (softer rate as the suspension compresses) and in this case it went from a rising rate (although small) to a falling rate. If the motion ratio decreases with increasing wheel travel, the wheel rate continues to fall through its travel. If the wheel rate is decreasing as the chassis rolls it can have a negative effect on handling as the suspension is actually getting softer as the chassis rolls. The opposite side of the car could, at the same time, possibly be getting stiffer which moves the roll center to the stiffer side.

Disclaimer

I hope this discussion helps with your understanding of how springs, spring rates, shock damping and forces are affected by your choice of component locations and motion ratios. This tutorial is provided as information to ISMA members and is not claimed to be used as recommendations for designs.